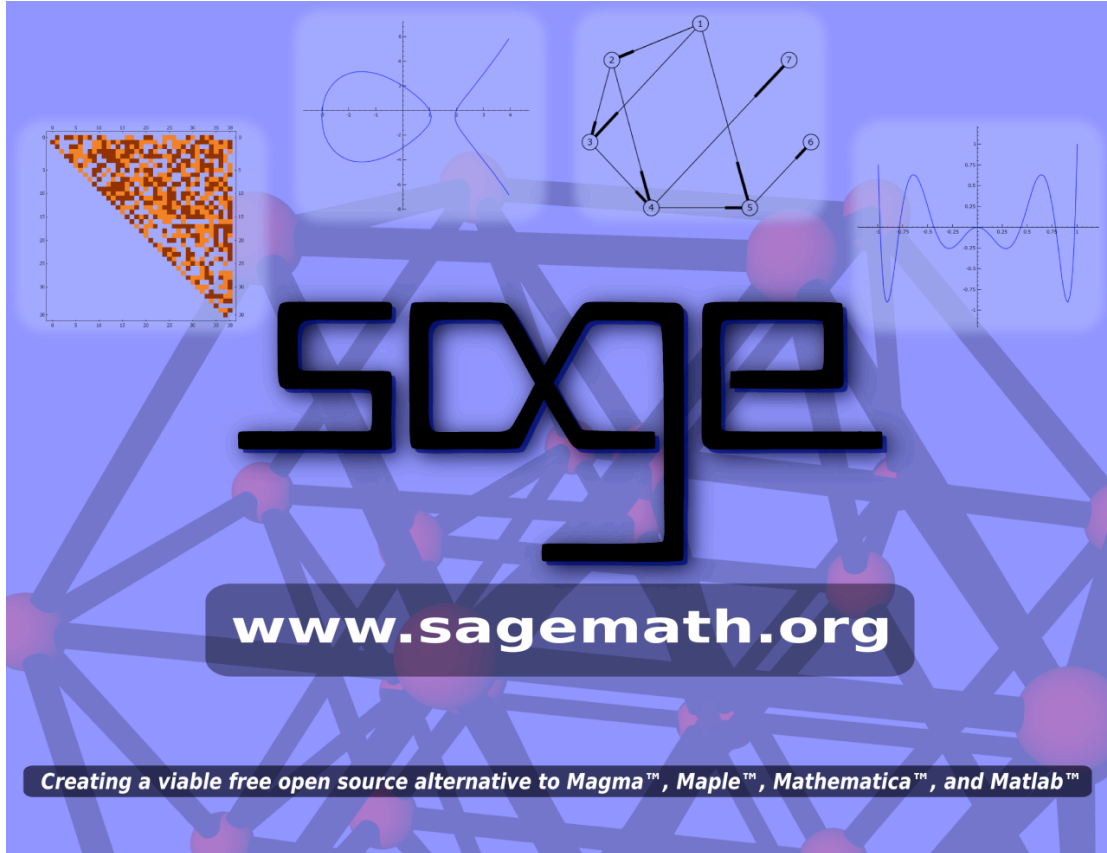


Boeing - June 12, 2008-stein

Sage: Open Source Mathematical Software



2 + 3

5

Features! Features! Features!

Sage can do *a lot*.

1. Numerical linear algebra, optimization (numpy, scipy, gsl)
2. Exact linear and commutative algebra (Linbox, IML, etc.)
3. Group theory
4. Number theory
5. Symbolic calculus
6. Coding theory
7. Cryptography and cryptanalysis
8. Graph theory
9. Combinatorics
10. 2d and 3d plotting
11. Statistics (Sage includes R)
12. Fast compiled code
13. Use Fortran in the notebook
14. Everything is 100% open source and free (just like Linux), so you can **integrate** it into your research environment however you want and change everything.

[Reference Manual \(over 3000 pages, all new\)](#)

A Few Examples...

```
a = random_matrix(ZZ,200)
time a.determinant()
-891333571303546314190713149714365469854126639872288088189099465125
2952224924459616064752863102724856196313867237102680359260645402195
5619157625512881347758014703226397815191515766045749618376877637483
6115073880400316947967854237513489949299179171771542461691866009072
1660899110094353279931623094077742403844814799452304289642545400341
9728338126710617488140465528342971655392964422132474573159112945004
27072927513336401024689577674960135323992286833474185137
Time: CPU 0.24 s, Wall: 0.25 s
```

```
a = random_matrix(RDF,1000)
time a.LU()
```

```
(1000 x 1000 dense matrix over Real Double Field, 1000 x 1000 dense
matrix over Real Double Field, 1000 x 1000 dense matrix over Real
Double Field)
Time: CPU 1.67 s, Wall: 1.73 s
```

```
factor(9023809283409823094892038402834098230942304)
```

```
2^5 * 967 * 11339824399261558063 * 25716219628037168507
```

```
time p = number_of_partitions(10^8)
p.ndigits()
```

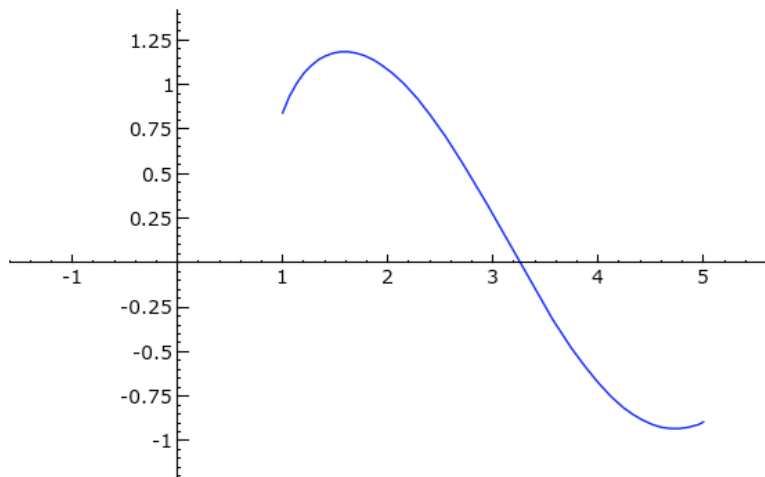
```
11132
```

```
CPU time: 3.83 s, Wall time: 3.95 s
```

```
var('x')
show( integrate(sin(x) + log(x)/x^2) )
```

$$\frac{-\log(x)}{x} - \cos(x) - \frac{1}{x}$$

```
plot(sin(x)+log(x)/x^2, 1,5)
```



```
# symbolic
a = log(2); a
log(2)
```

```
# numerical (straight call to c library)
a = math.log(2); a
0.69314718055994529
```

```
def class_hierarchy(cls, v):
    v.append(str(cls))
    for supercls in cls.__bases__:
        class_hierarchy(supercls, v)

@interact
def _(object=1):
    print '<html>'
```

```

print '<h1>Inheritance hierarchy of %r</h1>'%object
print '<font color="#333333"><pre>'
v = []
class_hierarchy(object.__class__, v)
print '\n'.join(['.'*(3*i)+w for i, w in
enumerate(reversed(v))]).replace('<', '<')
print '</pre></font></html>'

```

object

Inheritance hierarchy of 1

```

<type 'object'>
...<type 'sage.structure.sage_object.SageObject'>
.....<type 'sage.structure.element.Element'>
.....<type 'sage.structure.element.ModuleElement'>
.....<type 'sage.structure.element.RingElement'>
.....<type 'sage.structure.element.CommutativeRingElement'>
.....<type 'sage.structure.element.IntegralDomainElement'>
.....<type 'sage.structure.element.DedekindDomainElement'>
.....<type 'sage.structure.element.PrincipalIdealDomainElement'>
.....<type 'sage.structure.element.EuclideanDomainElement'>
.....<type 'sage.rings.integer.Integer'>

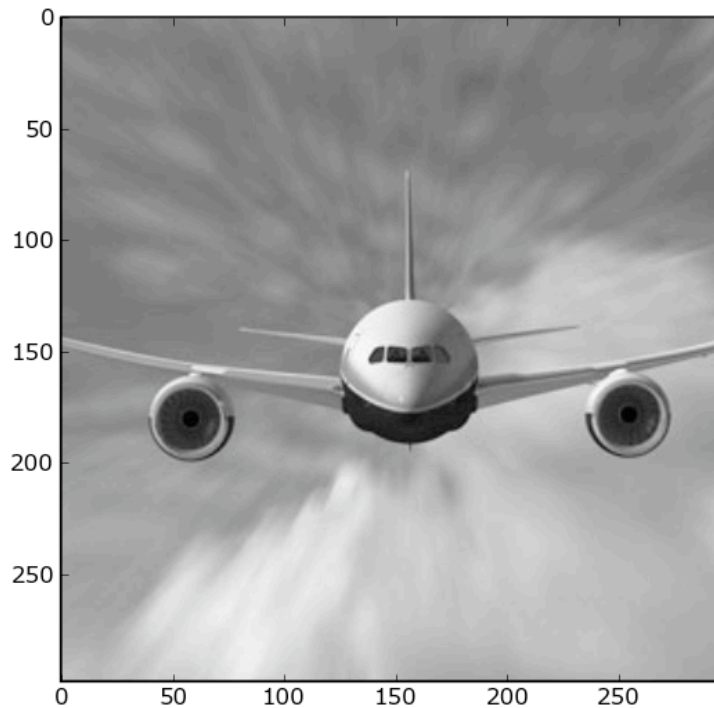
```

```

#auto
import pylab
A = pylab.imread(DATA + 'boeing.png')
Agray = pylab.mean(A,2)
pylab.imshow(Agray)
pylab.gray()
pylab.savefig('orig.png')

```

CPU time: 0.74 s, Wall time: 1.73 s

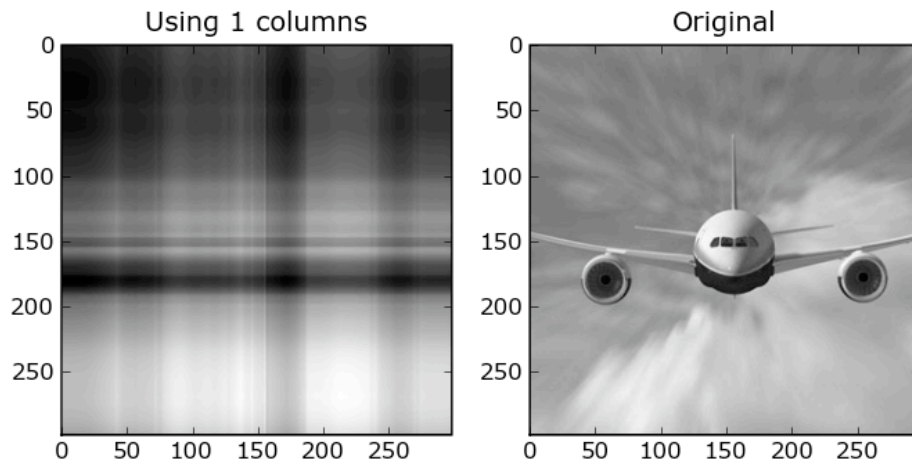


```

%hide #code to implement using SVD
def svd_image(Agray,i):
    siz = pylab.shape(Agray)
    u,s,v = pylab.linalg.svd(Agray)
    Anew = s[0]*pylab.outer(u[0:,0],v[0,0:])
    for j in range(i):
        uu = u[0:,j+1]; vv = v[j+1,0:]
        op = pylab.outer(uu,vv)
        Anew = copy(Anew) + pylab.dot(s[j+1],op)
    pylab.figure()
    pylab.subplot(121)
    pylab.imshow(Anew)
    pylab.gray()
    pylab.title('Using '+str(i+1)+ ' columns')
    pylab.subplot(122)
    pylab.imshow(Agray)
pylab.gray()
    pylab.title('Original')
pylab.savefig('orig.png')
    return Agray, Anew
@interact
def energy_in_svd(i=slider(0,100,1)):
    svd_image(Agray,i)

```

i



```
# Steiner surface/Roman's surface with embedded sphere
u, v = var('u,v')
fx = sin(2*u) * cos(v) * cos(v); fy = sin(u)*sin(2 * v)
fz = cos(u) * sin(2*v)
(sphere((0,0,0), 0.7, opacity=0.5) +
parametric_plot3d([fx, fy, fz], (u, -pi/2, pi/2), (v, -pi/2,pi/2),
frame=False, color="red"))
```

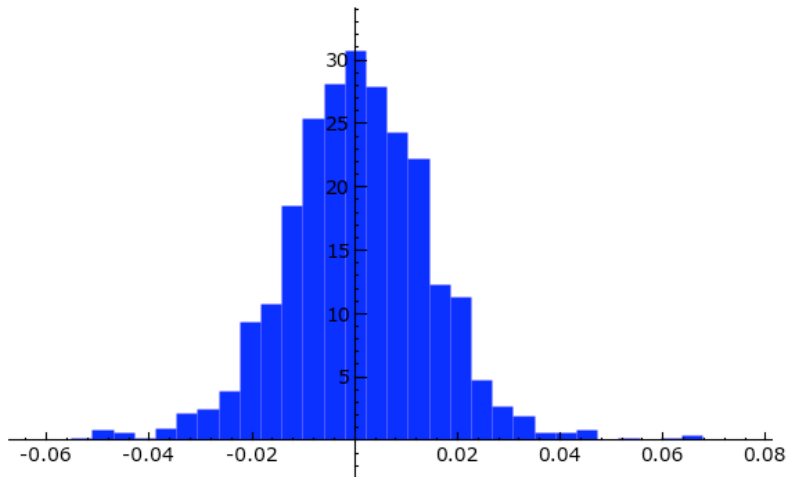
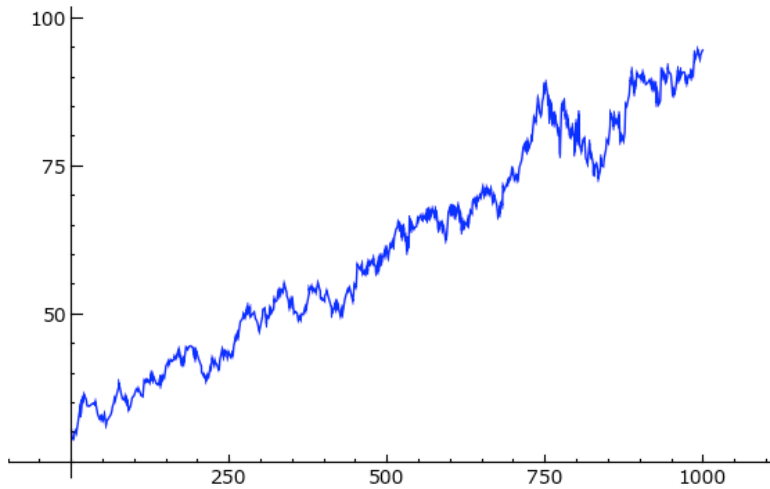
```
# Yoda! -- over 50,000 triangles.
from scipy import io
x = io.loadmat(DATA + 'yodapose.mat')
from sage.plot.plot3d.index_face_set import IndexFaceSet
V = x['V']; F3=x['F3']-1; F4=x['F4']-1
Y = IndexFaceSet(F3,V,color='green') +
IndexFaceSet(F4,V,color='green')
Y = Y.rotateX(-1)
Y.show(aspect_ratio=[1,1,1], frame=False, figsize=4)
```

```
#S = finance.Stock('ba').close()
S = load(DATA+'ba.sobj')
```

```
@interact
def _(days=(1300,(20..len(S)))):
    print "Last %s days of Boeing stock price and log diffs
    histogram"%days
    S[-days:].plot().show()
    S[-days:].log().diffs().plot_histogram(30).show()
```

days

Last 1277 days of Boeing stock price and log diffs histogram



Sage is Fast: Cython

Robert Bradshaw will talk more about this later...

```
def mymean(n):
    s = 0
    for m in range(n):
        s += m
```

```
return float(s) / n
```

```
time mymean(10^6)
```

```
499999.5
```

```
CPU time: 1.20 s, Wall time: 1.22 s
```

```
%cython
```

```
def mymean2(int n):
```

```
    cdef double s = 0
```

```
    cdef int m
```

```
    for m from 0 <= m < n:
```

```
        s += m
```

```
    return s / n
```

[Users wa... code_sage269_spyx.c](#)

[Users wa...age269_spyx.pyx.ht](#)

```
time mymean2(10^6)
```

```
499999.5
```

```
CPU time: 0.00 s, Wall time: 0.00 s
```

```
timeit('mymean2(10^6)')
```

```
625 loops, best of 3: 1.19 ms per loop
```

```
1.2/(1.2*10^(-3))
```

```
1000.0000000000000
```

```
v = finance.TimeSeries([1..10^6])
```

```
timeit('v.standard_deviation()') # straight Cython -- way faster  
than R/scipy/matlab...
```

```
125 loops, best of 3: 3.3 ms per loop
```

```
import scipy.stats
```

```
w = v.numpy()
```

```
timeit('scipy.stats.std(w)')
```

```
25 loops, best of 3: 35.1 ms per loop
```

You Can Use Java From Sage

```
from jpye import java, startJVM
```

```
startJVM('/System/Library/Frameworks/JavaVM.framework/Versions/1.5.0/L
```



```
java.lang.System.out.println('Hello Boeing!')
```

Hello Boeing!

This uses the JNI (Java Native Interface).

For more, see <http://jpye.sourceforge.net...>

History

- *I started Sage* at Harvard in January 2005, after working in computational number theory for nearly a decade using C++ and Magma.
- No existing mathematical software was technically good enough for what I **needed** for my research and teaching.
- Sage-1.0 released February 2006 at Sage Days 1 (San Diego).
- Sage Days 2, 3, 4, 5, 6, 6.5, 7, 8. At UCLA, UW, Cambridge, Bristol, Austin, etc. Sage DAys 9,10,11, 12 coming up fast.
- Funding from NSF, DoD, Microsoft, private donors, etc. About 100 contributors.



Sage from an Engineer's Perspective

- Sage does math **ALSO**
 - Sage is easily integrated with *anything*
 - Python was designed as a glue language
 - Open Source - Hack the code
 - Cython makes short work of wrapping C/C++ (and .o, .so, etc.)
-

Glue It All Together With Python

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Position paper for the [OMG-DARPA-MCC Workshop on Compositional Software Architecture](#) in Monterey, California, January 6-8, 1998.

Introduction

Python is an advanced scripting language that is being used successfully to glue together large software components. It spans multiple platforms, middleware products, and application domains. Python is an object-oriented language with high-level data structures, dynamic typing, and dynamic binding. Python has been around since 1991, and has a very active user community. For more information, see the Python website <http://www.python.org>.

Python

- "Python is fast enough for our site and allows us to *produce maintainable features in record times*, with a minimum of developers," said Cuong Do, Software Architect, [YouTube.com](#).
- "Google has made no secret of the fact they use Python a lot for a number of internal projects. Even knowing that, once *I was an employee, I was amazed at how much Python code there actually is in the Google source code system.*", said Guido van Rossum, [Google](#), creator of Python.
- "Python plays a key role in our production pipeline. Without it a project the size of *Star Wars: Episode II* would have been very difficult to pull off. From crowd rendering to batch processing to compositing, *Python binds all things together*," said Tommy Burnette, Senior Technical Director, [Industrial Light and Magic](#).