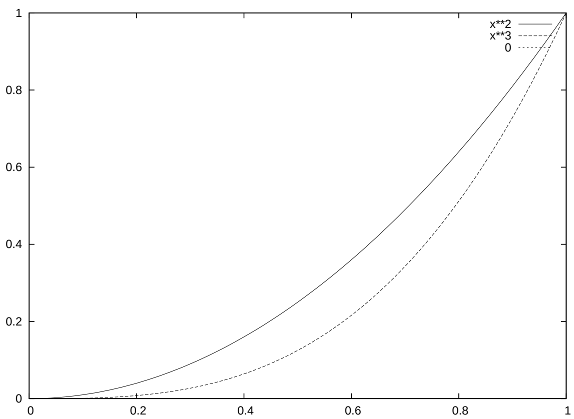
Today: Quiz!
Next: Polar coordinates, etc.
Questions:?
Recall: Find volume by integrating cross section of area. (draw picture)

Example 1.5.2. Find the volume of the solid obtained by rotating the following region about the x axis: the region enclosed by $y = x^2$ and $y = x^3$ between $x = 0$ and $x = 1$.

Figure 1.5.2: Find the volume of the flower pot



The cross section is a "washer", and the area as a function of x is

$$A(x) = \pi(r_o(x)^2 - r_i(x)^2) = \pi(x^4 - x^6).$$

The volume is thus

$$\int_0^1 A(x)dx = \int_0^1 \left(\frac{1}{5}x^5 - \frac{1}{7}x^7\right) dx = \pi\left[\frac{1}{5}x^5 - \frac{1}{7}x^7\right]_0^1 = \frac{2}{35}\pi.$$

Example 1.5.3. One of the most important examples of a volume is the volume V of a sphere of radius r. Let's find it! We'll just compute the volume of a half and multiply by 2. The cross sectional area is

$$A(x) = \pi r(x)^2 = \pi(\sqrt{r^2 - x^2})^2 = \pi(r^2 - x^2).$$

Then

$$\frac{1}{2}V = \int_0^r \pi(r^2 - x^2)dx = \pi\left[r^2x - \frac{1}{3}x^3\right]_0^r = \pi r^3 - \frac{1}{3}\pi r^3 = \frac{2}{3}\pi r^3.$$

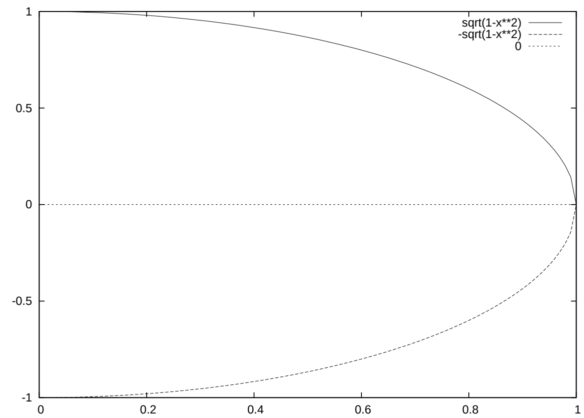
Thus $V = (4/3)\pi r^3$.

Example 1.5.4. Find volume of intersection of two spheres of radius r, where the center of each sphere lies on the edge of the other sphere.

From the picture we see that the answer is

$$2\int_{r/2}^r A(x),$$

Figure 1.5.3: Cross section of a half of sphere with radius 1



where $A(x)$ is *exactly* as in Example 1.5.3. We have

$$2\int_{r/2}^{r} \pi(r^2 - x^2)dx = \frac{5}{12}\pi r^3.$$