Math 168: Homework Assignment 5

William Stein

Due: Wednesday, Nov 2, 2005

The problems have equal point value, and multi-part problems are of the same value.

1 Problems

1. Suppose $\gamma = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$ is a matrix with real entries and positive determinant. Prove that if $z \in \mathbb{C}$ is a complex number with positive imaginary part, then the imaginary part of $\gamma(z) = (az+b)/(cz+d)$ is also positive.

2. (a) Prove that a polynomial is an analytic function on $\mathbb{C}$.
   (b) Prove that a rational function (quotient of two polynomials) is a meromorphic function on $\mathbb{C}$.

3. Suppose f and g are weakly modular functions with $f \neq 0$.
   (a) Prove that the product fg is a weakly modular function.
   (b) Prove that $1/f$ is a weakly modular function.
   (c) If f and g are modular functions, show that fg is a modular function.
   (d) If f and g are modular forms, show that fg is a modular form.

4. Suppose f is a weakly modular function of odd weight k. Show that $f = 0$.

5. (a) Prove that $\Gamma_1(N)$ is a group.
   (b) Prove that $\Gamma_1(N)$ has finite index in $\mathrm{SL}_2(\mathbb{Z})$ (Hint: it contains the kernel of the homomorphism $\mathrm{SL}_2(\mathbb{Z}) \to \mathrm{SL}_2(\mathbb{Z}/N\mathbb{Z})$.)