Mathematics 21b.
Linear Algebra and Differential Equations

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1. For each of the following, circle **T** if the statement is true, or **F** if the statement is false. In either case, justify your response.

   T **F** If $A^n = I_n$, then A must be invertible.

   T **F** If $A^2 = A$ for an invertible $n \times n$ matrix A, then $A = I_n$.

T **F** There exist a 2×3 matrix A and a 3×2 matrix B such that $AB = I_2$.

T **F** There exist a 3×2 matrix A and a 2×3 matrix B such that $AB = I_3$.

T **F** If $A^2 + 3A + 4I_3 = 0$ for a 3×3 matrix A, then A must be invertible.

T **F** If A and B are two 4×3 matrices such that $A\vec{v} = B\vec{v} \;\; \forall v \in \mathbb{R}^3$, then $A = B$.

T **F** If $A \in \mathbb{R}^{n\times n}$ has the property that $A^2 = 0$, then $\text{im}(A) \subset \ker(A)$.

T **F** If $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{n\times p}$, and $\ker(A) = \text{im}(B)$, then $AB = 0$.

T **F** If $A \in \mathbb{R}^{3\times 3}$ is a rotation matrix about the line spanned by $\vec{v}$, then $\ker(A - I_3) = \{c\vec{v} \ : \ c \in \mathbb{R}\}$.

2. Let $A \in \mathbb{R}^{n\times n}$ with the following property:

$$\sum_{i=1}^{n}(a_{ki}) = 0 \quad \forall k \in \{1, \dots n\}.$$

Prove or disprove: the matrix A is invertible.

3. Let A be a 3×5 matrix and B a 5×3 matrix with $AB = I_3$. Explain why $\text{rank}(A) = \text{rank}(B) = 3$.