
90a:20105

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Hecke operators and noncongruence subgroups.

Including a letter from J.-P. Serre.

Group theory (*Singapore*, 1987), 215–224, *de Gruyter*, *Berlin*, 1989.
20H25 (11F06)

This paper deals with the theory of Hecke operators on noncongruence subgroups (of finite index) in the group $\Gamma = \mathrm{SL}_2(\mathbf{Z})$. Let $G \le \Gamma$ be a subgroup of finite index and T_n^G the usual Hecke operator defined by averaging over $G/(G \cap M^{-1}GA)$, $M = \begin{pmatrix} n & 0 \\ 0 & 1 \end{pmatrix}$. The author studies the following perhaps surprising conjecture suggested by unpublished work of A. O. L. Atkin. Atkin's conjecture: If p is a prime and f a modular form on G of weight $k \ge 1$ then $f \circ T_p^G$ is a form on $\overline{G}$, where $\overline{G}$ is the intersection of all congruence subgroups of Γ which contain G.

The main result of the paper is to prove Atkin's conjecture in the case that the core of G, defined by $G_0 = \bigcap_{x\in\Gamma} G^x$, satisfies $\overline{G_0} = \Gamma$. The contribution of the author is to reduce this to the assertion that Atkin's conjecture holds for G_0 itself. The latter proof is provided by J.-P. Serre in a letter to the author (dated June 24, 1987) and appended to the paper, where in fact a sharper result is established: if $G \not\le \Gamma$ has finite index and $\overline{G} = \Gamma$ then $T_p^G = T_p^\Gamma \circ \mathrm{Tr}_G^\Gamma$ (where TR_G^Γ is the usual trace map from forms on G to forms on Γ). Serre's proof is quite ingenious, and makes use of a result of J. L. Mennicke [Invent. Math. **4** (1967), 202–228; MR **37** #1485] which states that, unlike Γ, every subgroup of $\mathrm{SL}_2(\mathbf{Z}[1/p])$ of finite index is a congruence subgroup.

{For the entire collection see 89j:20001}

Reviewed by *Geoffrey Mason*