Math 581e, Fall 2012, Homework 6

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Due: Friday, November 9, 2012

There are 4 problems. Turn your solutions in Friday, November 9, 2012 in class. You may work with other people and can find the LaTeX of this file at http://wstein.org/edu/2012/ant/hw/. If you use Sage to solve a problem, include your code in your solution. I have office hours 12:30–2:00 on Wednesdays in Padelford C423.

1. Explicitly determine the class group of $\mathbb{Q}(\sqrt{23})$ by computing the Minkowski bound, enumerating ideals, etc., like I did with $\mathbb{Q}(\sqrt{10})$ in class. Show as many details as you can. Also, double check your answer by computing the class group in Sage and comparing results.
2. It is generally conjectured (and easy to be convinced from data) that there are infinitely many quadratic fields of class number 1, though nobody knows how to prove this. Do you think there are infinitely many *cubic* number fields of class number 1? Give computational evidence for your conjecture.
3. Let $\mathcal{O}_K$ be the ring of integers of a number field. If $\alpha \in \mathcal{O}_K$ with $\mathbb{Z}[\alpha]$ of finite index in $\mathcal{O}_K$, and f is the minimal polynomial of α, prove that $\mathrm{Disc}(f) = \mathrm{Disc}(\mathbb{Z}[\alpha])$. [To see this, note that if we choose the basis $1, \alpha, \ldots, \alpha^{n-1}$ for $\mathbb{Z}[\alpha]$, then both discriminants are the square of the same Vandermonde determinant.]
4. Provide a status update on your final project (which is due **December 7**).
   (a) What is the latest title/abstract?
   (b) How far along have you got with it? (How many hours left until it is polished and done?)
   (c) Any questions for me about it?