Math 581e, Fall 2012, Homework 5

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Due: Friday, November 2, 2012

There are 4 problems. Turn your solutions in Friday, November 2, 2012 in class. You may work with other people and can find the LaTeX of this file at http://wstein.org/edu/2012/ant/hw/. If you use Sage to solve a problem, include your code in your solution. I have office hours 12:30–2:00 on Wednesdays in Padelford C423.

1. Let I, J be nonzero ideals in a Dedekind domain R. Prove that if $I \cap J = IJ$, then I and J are coprime (i.e., $I + J = 1$).

2. Let I, J, K be nonzero ideals in a commutative ring R. Prove that $(I + J)K = IK + JK$.

3. Find in any way you want an element $\alpha \in \mathbb{Q}(\sqrt[3]{7})$ such that

$$\alpha \equiv 0 \pmod{(2, \sqrt[3]{7}+1)}, \qquad \alpha \equiv 1 \pmod{(2, \sqrt[3]{7}^2+\sqrt[3]{7}+1)}, \qquad \alpha \equiv \sqrt[3]{7} \pmod{(3, \sqrt[3]{7}-1)}.$$

4. Consider the ideal $I = (2(\sqrt[3]{7} + 1), \sqrt[3]{7}^2 + 2\sqrt[3]{7} + 1)$ in the ring of integers of $K = \mathbb{Q}(\sqrt[3]{7})$. Find an element $\alpha \in \mathbb{Q}(\sqrt[3]{7})$ such that $I = (32\sqrt[3]{7}, \alpha)$.