Stochastic Financial Mathematics

Exchange Rates and Volatility

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Introduction

For this project, we used the Sage computational mathematics platform to manipulate currency data in order to apply a variant of the Black-Scholes model to determine the volatility of various currencies over time. The goal was to determine, by means of analyzing currency data, times of high economic volatility (crises) in various economical regions.

One of the main goals of this investigation was to show that a crisis occurred in late 2008.

Five currencies were considered for the project: the Euro, the UK Pound, the Chinese Yuan, the Japanese Yen, and the Russian Ruble.

This paper deals with the process and analysis of the project. To see the Sage worksheet used to manipulate the data, see http://480.sagenb.org/home/pub/130/

Method

First, define an *n*-long set *S* of data points containing the exchange value of a currency. We begin our analysis by stating that that currency exchange rates are a stochastic process instead of a deterministic process. That is, the exchange rate on one day does not determine the exchange rate on the next, but there is a relationship between the two which involves a random variable.

In other words, we can say that today's exchange rate, S_k , is related to the exchange rate for the next day, S_{k+1} , as follows: $S_{k+1}=S_k e^{x_{k+1}}$ for some x_{k+1} . We can equivalently say that $x_{k+1}=\ln(\frac{S_{k+1}}{S_k})$.

At this point, we assume that $x_0 \dots x_{n-1}$ are independent identically distributed random variables, with a Normal(μ, σ) distribution. Call the (*n*-1)-long set the *X* set. Due to this assumption, we know that each x_k has a variance. We know further that we are able to calculate the empirical variance of a set of x_k 's. We also know that with a large enough data set, the empirical variance of x_k approaches the actual variance.

Due to the way we have stated our assumptions, the empirical variance represents the fluctuation, or volatility, of the exchange rate. If the volatility of the exchange rate is high, that means the exchange rate is fluctuating rapidly and unpredictably. Additionally, we further assume that if the exchange rate has a high volatility, the associated market has entered a state of crisis. A small volatility means that the exchange rate is changing slowly, and that the associated market is stable.

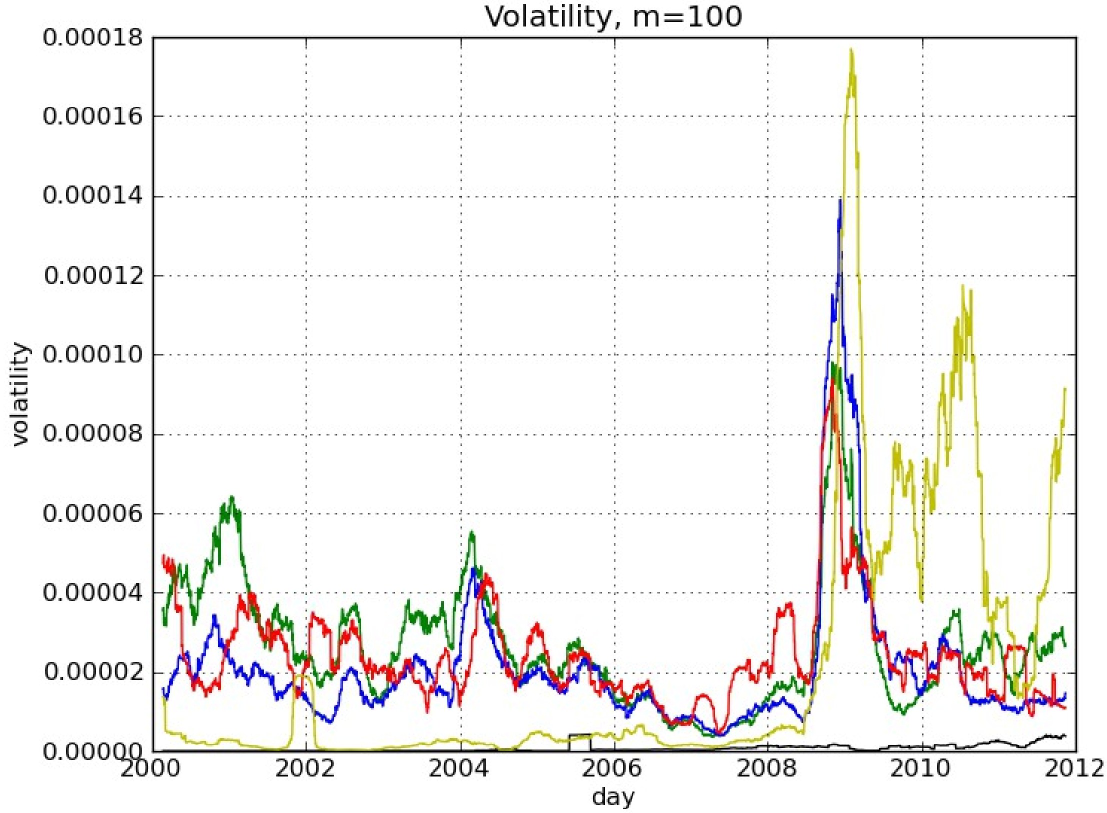
In order to calculate the empirical variance for a value x_k, a set of data points is required. This data will be a subset of the *X* set. We will calculate the empirical variance of an x_k by considering the subset $x_{k-\frac{m}{2}} \dots x_{k+\frac{m}{2}}$. That is, we will take a window of size *m* and calculate its empirical variance, and take that to be the empirical variance of the midpoint of that subset, x_k.

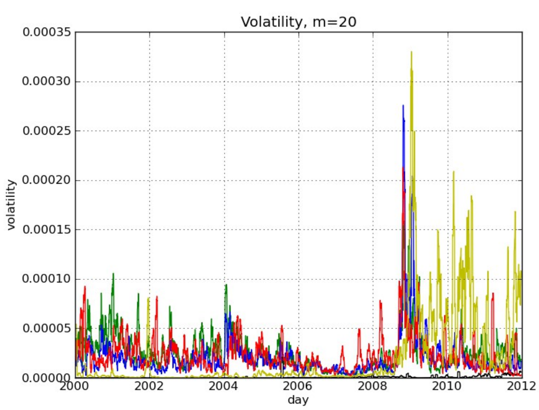
Our data, the *S* set, has a single data point for each day. Therefore, in this case, the window *m* represents a calculation of the empirical variance over *m* days. Special care must be taken when choosing the size of the window. A large *m* may provide a very broad and possibly meaningless result, while a small *m* may provide an empirical variance with a large error. Recall that the precision of statistical estimates is roughly $\frac{1}{\sqrt{M}}$ where *M* is the quantity of data. Choosing $M=m=100$ gives us a precision of 0.1, which means we have a good estimate. This means we are considering the empirical variance of a currency using just over 3 months of data. More data points would lead to an empirical variance calculated over a smaller real-time period, and may lead to more detailed analysis.

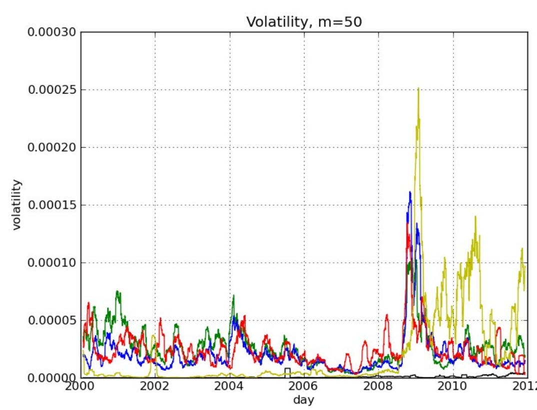
When calculating the empirical variance, please note that we are calculating a moving variance. That is, we calculate $Var[x_k]$ using $x_{k-\frac{m}{2}} \dots x_{k+\frac{m}{2}}$ and then we calculate $Var[x_{k+1}]$ using $x_{k-\frac{m}{2}+1} \dots x_{k+\frac{m}{2}+1}$. Like a moving average, this provides more data points and creates a smoother graph of results. We begin the calculation at $x_{\frac{m}{2}}$ and terminate it upon reaching $x_{n-1-\frac{m}{2}}$. Note that when starting with *n* values in *S*, we end up with *n*-*m*-1 empirical variances.

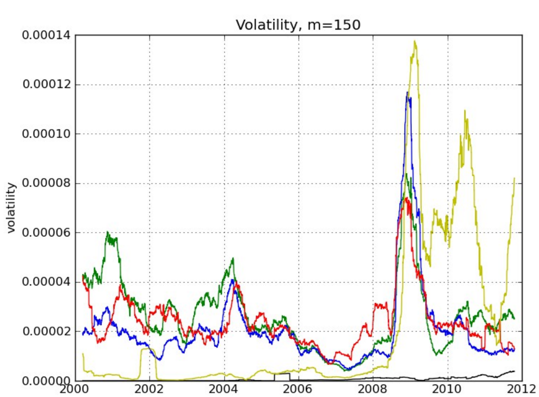
Since changing the window size is trivial due to the way the calculations were performed in Sage, we have provided data charts with varying window sizes for consideration.

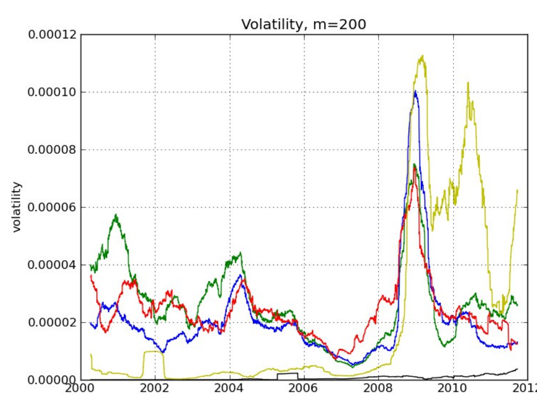
Euro-green, UK Pound-blue, Chinese Yuan-black, Japanese Yen-red, Russian Ruble-yellow

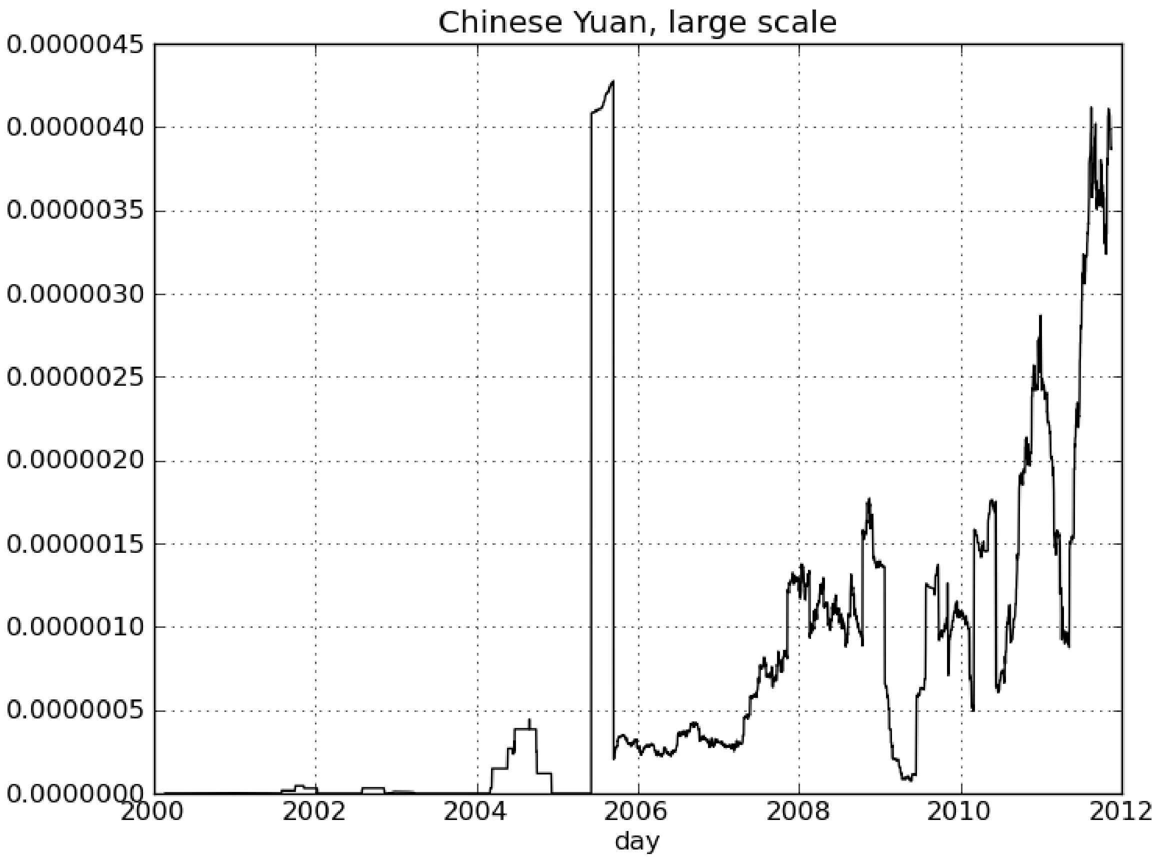












Analysis of Data

From the plot of the variances, we are able to draw four major observations:

1. The 2008 market crisis[1] is readily visible.
2. The volatility of the Chinese Yuan is an order of magnitude smaller than the other currencies considered.
3. The volatility of the Euro, Pound, and Yen seem to be correlated, especially from 2005-2007.
4. The Russian Ruble is highly volatile following the 2008 crisis.

As observation 1, the visibility of the 2008 crisis, was a primary goal for this project, we can be more certain of the usefulness of the model.

Observation 2, the low volatility of the Yuan, can be explained by noting that the Chinese

1 Simkovic, Michael. (2009). Secret Liens and the Financial Crisis of 2008. *American Bankruptcy Law Journal*, 83, 253. Retrieved from http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1323190

Yuan operates under a fixed rate regime[2]. A change in the regime was adopted by the Chinese government in 2005[3] which is clearly reflected in the detailed plot of the Yuan's variance.

Observation 3, the seeming correlation of the Euro, Pound, and Yen, perhaps warrants further investigation, but may be explained by the inter-relation of the three economies.

Observation 4, the high volatility of the Ruble, can be explained by the volatility of energy prices, as the Russian economy has large sectors based on the production and export of energy[4].

Conclusion

Using a stochastic model for exchange rates can provide a volatility metric which can contribute to understanding the stability of national economies.

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2 (2009, November 19). China's exchange-rate policy: A yuan-sided argument: Why China resists foreign demands to revalue its currency. *The Economist*. Retrieved from http://www.economist.com/node/14921327

3 Goodman, Peter S. (2005, July 22). China Ends Fixed-Rate Currency. *The Washington Post*. Retrieved from http://www.washingtonpost.com/wp-dyn/content/article/2005/07/21/AR2005072100351.html

4 Wu, Yu-Shan. (2010). Russia and the CIS in 2009: Pillar of the System Shaken. *Asian Review*, 50, 76-88. Retrieved from http://www.jstor.org/stable/10.1525/as.2010.50.1.76