

Math 581g, Fall 2011, Homework 3

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Due: Monday, October 24, 2011

There are 3 problems. Turn your solutions in Wednesday, October 26, 2011 in class (or via email). (WARNING: I will be gone on Monday, October 24, 2011 and Friday, October 28, 2011 to give two talks in Santa Cruz, California and one in Austin, Texas but the postdocs will come to class and answer any questions you can think of.) You may work with other people and can find the L^AT_EX of this file at <http://wstein.org/edu/2011/581g/hw/>.

1. (Warm up) Prove that for τ in the upper half plane, we have

$$\overline{(e^{-2\pi i\bar{\tau}})} = e^{2\pi i\tau}.$$

2. Let d and m be positive integers. Prove that $\frac{1}{d} \sum_{b=0}^{d-1} (e^{\frac{2\pi im}{d}})^b$ is only nonzero if $d \mid m$, in which case the sum equals 1.
3. For $\tau \in \mathbb{C}$, let $E_\tau = \mathbb{C}/(\mathbb{Z}\tau + \mathbb{Z})$. Show that $\#\text{Aut}(E_i) = 4$, $\#\text{Aut}(E_\rho) = 6$, and $\#\text{Aut}(E_{\sqrt{-2}}) = 2$. (Recall that $\rho = e^{2\pi i/3}$.)