Math 480a: Sage, Spring 2011, Takehome Midterm

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Due May 16, 2011

Instructions: Do the following 5 problems, and turn them in by the beginning of class on Monday, May 16, 2011, as a Sage worksheet (email it to `math480spr11@gmail.com`). Your worksheet should be formatted as cleanly as possible; in particular, use formatted text cells instead of just dumping all of your answers in the executable cells. Each problem is worth 4 points. **You may not get interactive help on any of these problems from any other person, though use of Internet searches, documentation, etc., is encouraged. Cite sources.**

(Problem idea contributed by Alex.) Say that a positive integer n *digit divides itself* if every digit of n in base 10 divides n. For example, 128 digit divides itself since 1, 2, and 8 all divide 128 evenly. We'll say that 0 does not divide any positive integer, so a number with a digit of 0 never digit divides itself, thus 120 does not digit divide itself. Further examples: 36 digit divides itself, but 37 does not.

1. Write a function in Sage that takes in an integer and outputs `True` if that integer digit divides itself, and `False` otherwise. You should document your function well with a docstring (including INPUT/OUTPUT/EXAMPLES blocks) and comments (see the point example at `http://sagemath.org/doc/developer/conventions.html`), format your code nicely, and your function should handle bad input gracefully. In particular, if the user inputs something that can't be converted to an integer, a `TypeError` exception should be raised, and if the input is a 0 or negative, your code should raise a `ValueError`.

2. How many integers between 1 and 2000, inclusive, i.e., in `[1..2000]`, digit divide themselves?

3. Use whatever code you can write (possibly using Cython or `@parallel`) so that when you run the code, it computes the answer to the following question in less than 1 minute: *How many numbers less than* 10^8 *digit divide themselves?* [Fact: It is possible to write Cython code that when run answers this question in about 10 seconds CPU time.] *Do not just compute the answer using hours of compute time, then make a stupid function that just returns the answer; I mean that you have to write code to compute the answer for* 10^8 *using some algorithm, which also works for other input. Also, please provide timing of your code in a Sage cell, to save me from having to rerun your code.*

4. Define a function

   $$d(x) = \#\{\text{positive integers } n \le x \text{ that digit divide themselves}\}.$$

   For example, we have

   $$d(20) = \#\{1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 15\} = 12.$$

   (a) Draw a plot of $d(x)$ for $1 \le x \le 100$. This should look roughly like an irregular staircase. (Don't worry about trying to make each little stairstep vertical...)

   (b) Draw a plot of $d(x)$ for $1 \le x \le 10^4$, which should look much smoother.

   (c) Draw a plot of $d(x)$ for $1 \le x \le 10^5$, which should look surprisingly similar to the plot up to 10^4.

5. Find a page at `http://oeis.org/` that is about the above sequence of numbers, and use whatever you find to doublecheck whatever you can in the answers you gave above. (Your answer to this part of the problem should simply be a URL and any discussion of how you used the page as a double check to your work.)