

Math 480a: Sage, Spring 2011, Homework 7

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Due May 18, 2011

Instructions: Do the following 2 problems, and turn them in by the beginning of class on Wednesday, May 18, 2011. If you get help from a classmate or friend on any homework problem, please explicitly thank them in your write up.

- (a) Use Sage to compute the prime factorizations of every year from 2000 until 2500, inclusive. (Include your Sage code with your solution.)
(b) Which year has the most distinct prime factors?
- (a) Verify the Riemann Hypothesis bound for each *integer* $x = 3, 4, \dots, 1000$. (Your answer should be Sage code that does this verification; you might optionally want to draw a plot.) You will recall from class that this is the statement that

$$|\pi(x) - \text{Li}(x)| \leq \sqrt{x} \cdot \log(x). \quad (1)$$

Here, $\pi(x) = \#\{p : p \leq x \text{ is prime}\}$, which can be computed in Sage using the function `prime_pi`, and $\text{Li}(x) = \int_2^x \frac{1}{\log(t)}$, which can be computed in Sage using `Li`.

- (b) (*) Verify (via some possibly “mathematical” thinking) that (1) holds for all real numbers $3 \leq x \leq 1000$? [NOTE: You will loose or get no points for this part of the problem. It’s worth 0 points. Skip it if you don’t know how to solve it.]