Math 583: Computing With Modular Forms (Spring 2006)

MWF 2:30-3:20 in Smith 307
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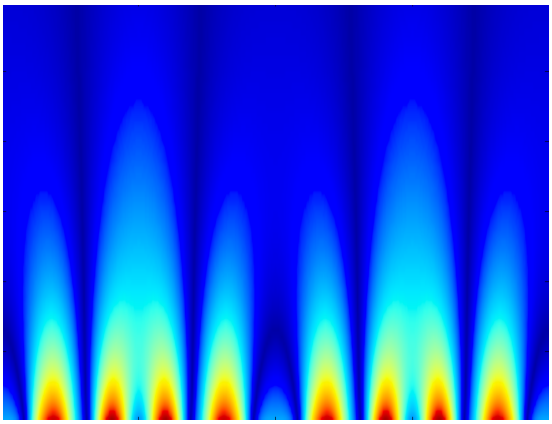
Office Hours: MW 1:30-2:20 in Paddelford 423
Website: http://modular.ucsd.edu/583 (this will change)

Abstract

According to Eichler there are five fundamental operations in arithmetic: "addition, subtraction, multiplication, division, and modular forms". This course is an introduction to modular forms with a special emphasis on how to compute with them. It is aimed at first-year graduate students.



1 Textbooks

The main text is my book *Computing With Modular Forms*. You can download a complete (but unpolished) version of the book from the course website. *This book will be published by the American Mathematical Society; the goal is to finish it by June 1, so it will be in print in time for the joint AMS meetings in January.* Four student projects from when I taught similar courses at Harvard and UCSD will be part of this book, and maybe your project will be too if you take this course.

Most of the algorithms in the book are implemented as part of the completely open source and free program SAGE: Software for Algebra and Geometry Experimentation. See `http://modular.ucsd.edu/sage/` for more details.

2 Course Outline

We will systematically work through the course textbook, skipping some topics because of lack of time (the book was written for a semester course).

1. Basic definitions
2. Modular forms of level 1

3. Modular forms of higher weight and level
4. Drawing pictures of modular forms
5. Eisenstein series, Bernoulli numbers, and Dirichlet characters
6. Computing modular forms using modular symbols
7. Special values of L-functions
8. Congruences and generators for Hecke algebras

3 Prerequisites

- *Complex analysis:* series, products, linear fractional transformations.
- *Algebraic number theory:* galois groups, cyclotomic fields, arithmetic in number fields.
- *Riemann surfaces:* differentials and homology.
- *Computer programming:* may be helpful for some of the exercises. We'll use SAGE/Python, which is easy to learn if you know any other language.

Don't worry if you are missing some of the prerequisites—talk to me.

4 Grading

Your grade will be some shade of "A", i.e., between 3.8 and 4.0. It will be based on:

- Weekly homework assignments, and
- A project that will either appear or contribute somehow to the published version of the course book or to SAGE.

There will be no exams.

5 Homework

There will be one homework assignment per week. It will be assigned by Wednesday, and be due the following Wednesday.

6 MSRI Workshop

I am organizing a 2-week MSRI workshop on Computing with Modular Forms in Berkeley, CA July 31–August 11, 2006. This workshop is aimed at graduate students and will have around 40 participants. If you are interested in attending, please let me know!