

# Homework 7: Class Groups and Elliptic Curves

DUE WEDNESDAY, NOVEMBER 14

William Stein

**Math 124** HARVARD UNIVERSITY **Fall 2001**

1. (10 points) For any negative discriminant  $D$ , let  $C_D$  denote the finite abelian group of equivalence classes of primitive positive definite quadratic forms of discriminant  $D$ . Use the PARI program `forms.gp` from lecture 24 (download it from my web page) to compute representatives for  $C_D$  and determine the structure of  $C_D$  as a product of cyclic groups for each of the following five values of  $D$ :

$$D = -155, -231, -660, -12104, -10015.$$

2. (6 points) Draw a beautiful graph of the set  $E(\mathbb{R})$  of real points on each of the following elliptic curves:
  - (i)  $y^2 = x^3 - 1296x + 11664$ ,
  - (ii)  $y^2 + y = x^3 - x$ ,
  - (iii)  $y^2 + y = x^3 - x^2 - 10x - 20$ .
3. (4 points) A rational solution to the equation  $y^2 - x^3 = -2$  is  $(3, 5)$ . Find a rational solution with  $x \neq 3$  by drawing the tangent line to  $(3, 5)$  and computing the third point of intersection.