

SACNAS

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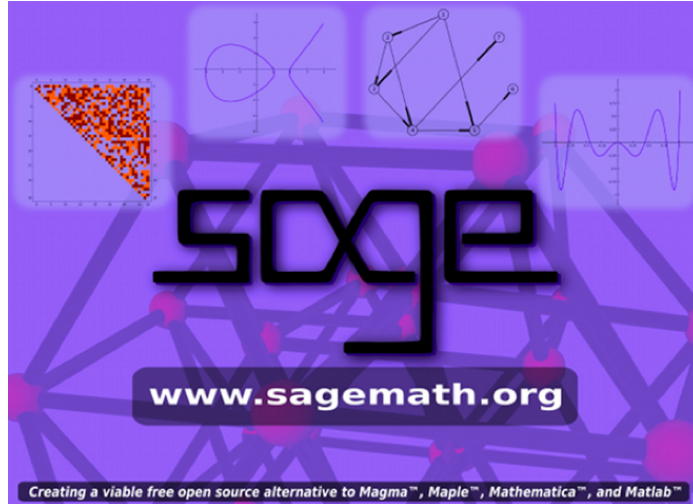
```
1+2+3 # testing
```

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Sage: Creating a Viable Open Source Alternative to Magma, Maple, Matlab, and Mathematica

SACNAS 2012, October 10, 2012

William Stein (Professor of Mathematics, University of Washington)



Sage: Mission Statement

"Create A Viable Free Open Source Alternative to Magma, Maple, Mathematica, and Matlab"

- **Mathematical features:** Of Magma, Maple, Mathematica, and Matlab, with comparable speed
- **Graphics:** 2d and 3d
- **Notebook:** Interactive graphical user interface
- **Documentation:** Books, papers, curriculum, etc.

Sage is not a drop-in replacement: does not run programs written in the custom languages of the Ma's.

Sage is not like Octave (versus Matlab).

Sage's culture, architecture, programming language, and feel are different than the Ma's.

Why *not* Magma, Maple, Matlab, Mathematica?

1. **Commercial:** Expensive for my collaborators and students. Not community owned.
2. **Closed:** Implementation of algorithms often secret
3. **Frustrating:** Tight control of development
4. **Copy protection:** Hard to run on supercomputer or my new laptop or after my 1-year license expires.
5. **Programming language:** All use a special math-only language
6. **Bugs:** Bug tracking is secret
7. **Compiler:** Lack of compilers for their math-only languages



1. **Python:** Sage uses a mainstream general purpose programming language (with a compiler: Cython)
2. **Distribution:** about 100 open source packages (**written by you** and your colleagues!)
3. **Interfaces:** smoothly tie together all these libraries and packages
4. **New library:** implements novel algorithms; over a half million lines; worldwide community of several hundred people.

```
# Sage uses Python code

def foo(n, m):
    if n > m:
        print("%s is bigger!"%n)

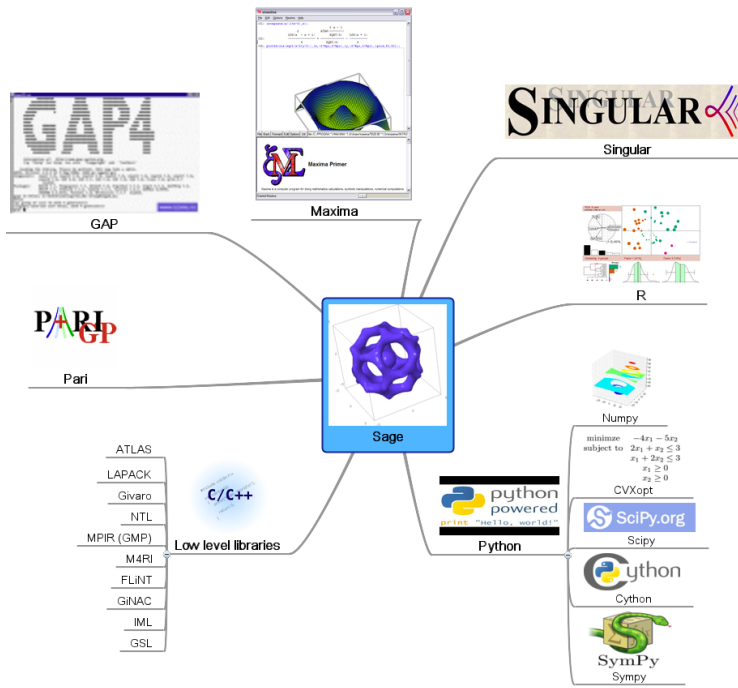
foo(10, 5)
```

```
10 is bigger!
```

```
foo('william', 'jon')
```

```
william is bigger!
```

Distribution



... each with their own rich history!



Hundreds of Sage Developers

(There are at least 247 contributors in 167 different places from all around the world.)



William Stein, Tim Abbott, Michael Abshoff, Antti Ajanki, Martin Albrecht, Nick Alexander, Bill Allombert, Ethan Van Andel, Ivan Andrus, Pablo Angulo, Benjamin Anteau, André Aplitzsch, Maite Aranes, Oscar Gerardo Lazo Arjona, Eviatar Bach, Jenniffer Balakrishnan, Jason Bandtow, Gregory Bard, Sébastien Barthélemy, Rob Beezer, Karim Belabas, Arnaud Bergeron, Luis Berloz, Erin Beyerstedt, Francois Bissey, Jonathan Bober, Tom Boothby, Nicolas Borie, Johan Bosman, Robert Bradshaw, Michael Brickenstein, Nils Bruin, André-Patrick Bubei, Stanislav Buljgin, Dan Bump, Itikhar Burnanuddin, Paul Butler, Oriol Castejón, Ondrej Certik, Wilson Cheung, Dan Christensen, Craig Citro, Anders Claesson, Francis Clarke, Timothy Clemans, Alex Clemesha, Nathann Cohen, Jenny Cooley, John Cremona, Karl-Dieter Crisman, Fidel Barrera Cruz, Doug Cutrell, Alyson Deines, Vincent Delecroix, Jeroen Demeyer, Tom Denton, Maarten Derick, Didier Deshommes, Ryan Dingman, Dan Drake, Tom Draper, Alexander Dreyer, Tim Dumol, Nathan Dunfield, Gabriel Ebner, Ben Edwards, Dana Ernst, Burcin Erocal, Ron Evans, Richard J. Fateman, Lars Fischer, Jean-Pierre Flori, Evan Fosmark, Laurent Fousse, Gary Furnish, Alex Ghitza, Andrzej Giniiewicz, Alain Giorgetti, Samuele Girardo, Amy Glen, Daniel Gordon, Chris Gorecki, Jan Groenewald, Rob Gross, Jason Grout, Ryan Grout, Mathieu Guay-Paquet, Alexey U. Gudchenko, Harold Gutch, Jonathan Gutw, Jose Guzman, Anna Haensch, Carlo Hamalainen, Marshall Hampton, Jon Hanke, David Møller Hansen, Mike Hansen, Bill Hart, David Harvey, Leif Hillie, Florent Hervet, Ryan Hinton, Neal Holz, Goham Mortza Hossain, Sean Howe, Alexander Hüfler, Wilfried Huus, Hamish Ivey-Law, Nazir Jaffery, Peter Jeremy, Peter Jipsen, Fredrik Johansson, Niles Johnson, Timo Jolivet, Benjamin Jones, David Joyner, Michael Kalweit, Josh Kantor, Kiran Kedlaya, Lloyd Kilford, Simon King, Keshav Kini, David Kirby, Emily Kirrman, David Kohel, Ted Kosan, Ross Kyprianou, Sébastien Labbé, Yann Laigle-Chapuy, Kwankyu Lee, Julien Leroy, Richard Lindner, David Loeffler, Miguel Marco, Michael Mardaus, Robert Mafik, Jason Martin, Alexandre Blondin Massé, Peter McNamara, Gregory McWhirter, Jason Merrill, Matthias Meulen, Robert Miller, Kate Minola, Moritz Minzlaff, Joel Mohler, Thierry Monteil, Peter Mora, Bobby Moretti, Rich Morin, Guillaume Moroz, Gregg Musiker, Tobias Nagel, Brett Nakashima, Pablo De Nápoli, Johan Sebastian Rosenkilde Nielsen, Minh Van Nguyen, Andrey Novoselisev, Christopher Olah, Johan Oudinet, Bill Page, Ronan Paixão, Willem Jan Palenstijn, John Palmieri, Dmitri Pasechnik, Javier López Peña, Paulo César Pereira de Andrade, David Perkinson, Clement Permet, John Perry, Pearu Peterson, David Poetsch-Helffer, Viviane Pons, Bill Purvis, Julien Puydt, Yi Qiang, Jordi Quer, Gustavo Rama, Jens Rasch, Martin Raum, Dorian Raymer, Stefan Reiterer, R. Rishikesh, David Roe, Bjarke Hammersholt Rourne, Gordon Royle, Serge A. Salamanka, Franco Salviola, Leonardo Sampaio, Kyle Schalm, Ed Scheinerman, Anne Schilling, Harald Schilly, Jack Schmidt, Michael Schneider, Christopher Schwan, Dag Sverre Seljebotn, Dan Shumow, Denis Simone, Steven Sivek, Nils-Peter Skrupupa, Jaap Spies, Jonathan Spreer, Armin Straub, Marco Streng, Kevin Stueve, Christian Stump, Blair Sutton, Chris Swierczewski, Luis Felipe Tabera Alonso, Glenn Tarbox, Philippe Theveny, Nicolas Thiery, Griffen Thoma, Emmanuel Thomé, John Thurber, Igor Tolkov, Gonzalo Tornaria, Kiminori Tsukazaki, Charlie Turner, Michel Vandenbergh, Joris Vankerschaver, Soledad Villar, John Voight, Felipe Voloch, Steve Vonn, Justin Walker, Mark Watkins, Georg S. Weber, Eric Webster, Ralf-Philipp Weimann, Joe Wetherell, Carl Witty, Cristian Wulthrich, Sorosh Yazdani, Dai S. Yu, Gary Zabrackis, Mike Zabrocki, Bin Zhang, Paul Zimmermann, Mao Ziyang

History

- **2005:** [SAGE-0.1 released February 1, 2005](#); SAGE=Software for Arithmetic Geometry Experimentation. Why did I name it "Sage"? claritalb.org
- **2006:** (2 Sage Days workshops); Sage is not just for number theory
- **2007:** (4 Sage Days) 100% test requirements; peer review of all new code ([see trac](#)); industry funding; NSF; Trophées du Libre
- **2008:** (7 Sage Days) Release managers besides me.
- **2009:** (8 Sage Days) Better foundations; 3d graphics; more developers (e.g., sage-combinat)
- **2010:** (13 Sage Days) More devs and users; nontrivial NSF grants
- **2011:** (12 Sage Days) Much faster <http://sagenb.org> with >50,000 accounts; very stable releases; undergrad curriculum development
- **2012:** (12 Sage Days)

- 2013: Release [Salvus...](#)

See [this article](#) for more details about the (pre-)history of Sage.

Random Question Break

????

Some random examples involving Sage:

```
factorial(3)
```

6

```
A = random_matrix(QQ, 4); b = A^(-1); b
```

```
[ 1 0 0 0]
[-27/34 5/17 -8/17 6/17]
[ 8/17 -8/17 6/17 4/17]
[ 11/34 3/17 2/17 7/17]
```

```
show(b)
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -\frac{27}{34} & \frac{5}{17} & -\frac{8}{17} & \frac{6}{17} \\ \frac{8}{17} & -\frac{8}{17} & \frac{6}{17} & \frac{4}{17} \\ \frac{11}{34} & \frac{3}{17} & \frac{2}{17} & \frac{7}{17} \end{pmatrix}$$

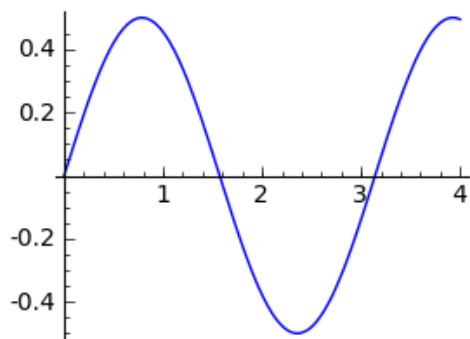
```
latex(b)
```

```
\left(\begin{array}{rrrr}
1 & 0 & 0 & 0 \\
-\frac{27}{34} & \frac{5}{17} & -\frac{8}{17} & \frac{6}{17} \\
\frac{8}{17} & -\frac{8}{17} & \frac{6}{17} & \frac{4}{17} \\
\frac{11}{34} & \frac{3}{17} & \frac{2}{17} & \frac{7}{17}
\end{array}\right)
```

```
show(integrate(sin(x)*cos(x), x))
```

$$-\frac{1}{2} \cos(x)^2$$

```
G = plot(sin(x)*cos(x), (0, 4), figsize=3)
G
```



```
G.save('a.pdf')
```

[a.pdf](#)

```
a = octave('rand(5)'); a
```

```
0.978093 0.232216 0.576728 0.74009 0.944403
0.0451772 0.82081 0.472989 0.0934609 0.537574
0.996148 0.66691 0.285855 0.513835 0.452995
0.999277 0.782493 0.668954 0.906361 0.711775
0.903178 0.192995 0.836577 0.306625 0.130191
```

```
a.eig()
```

```
(3.00683,0)
(-0.431469,0.298407)
(-0.431469,-0.298407)
(0.660756,0)
(0.316665,0)
```


How to Use Sage

- Type Python code into the boxes and press shift-enter or click "evaluate". [More about the notebook...](#) (read the bottom of the page that appears)
- There is [extensive documentation](#), including a [massive reference manual](#).
- Get help at: [live chat](#), ask.sagemath.org, and the [sage-support](#) google group.

Interactive Image Compression

(using [numpy](#))

```

import pylab, numpy

X = pylab.imread(DATA + 'sacnas.png')
A_image = numpy.mean(X, 2)
u,s,v = numpy.linalg.svd(A_image)
S = numpy.zeros(A_image.shape)
S[:len(s),:len(s)] = numpy.diag(s)
n = A_image.shape[0]

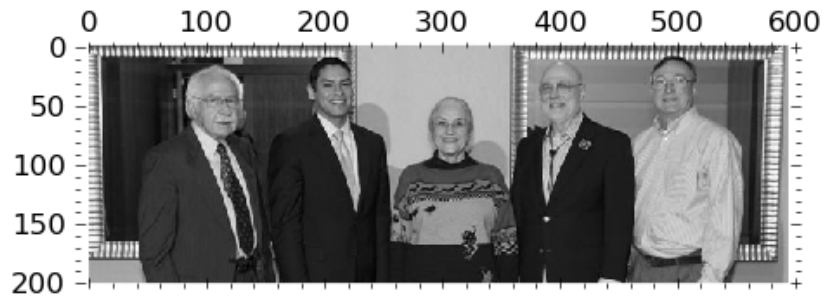
@interact
def svd_image(i = ("Eigenvalues (quality)",
                  (20,(1..A_image.shape[0]//2)))):
    A_approx = numpy.dot(numpy.dot(u[:, :i], S[:i, :i]), v[:i, :])
    g = graphics_array([matrix_plot(A_approx),
                        matrix_plot(A_image)])
    show(g, axes=False, figsize=8)
    html("%sx%s image compressed to %.1f%% of size using %s eigenvalues."%(
        A_image.shape[0], A_image.shape[1],100*(2.0*i*n+i)/(n*n), i))

```

Eigenvalues (quality)

20

200x593 image compressed to 8.0% of size using 20 eigenvalues.



Number Theory

```
factor(2009201020112012) # makes use of PARI
```

```
2^2 * 43 * 2269 * 5148259709
```

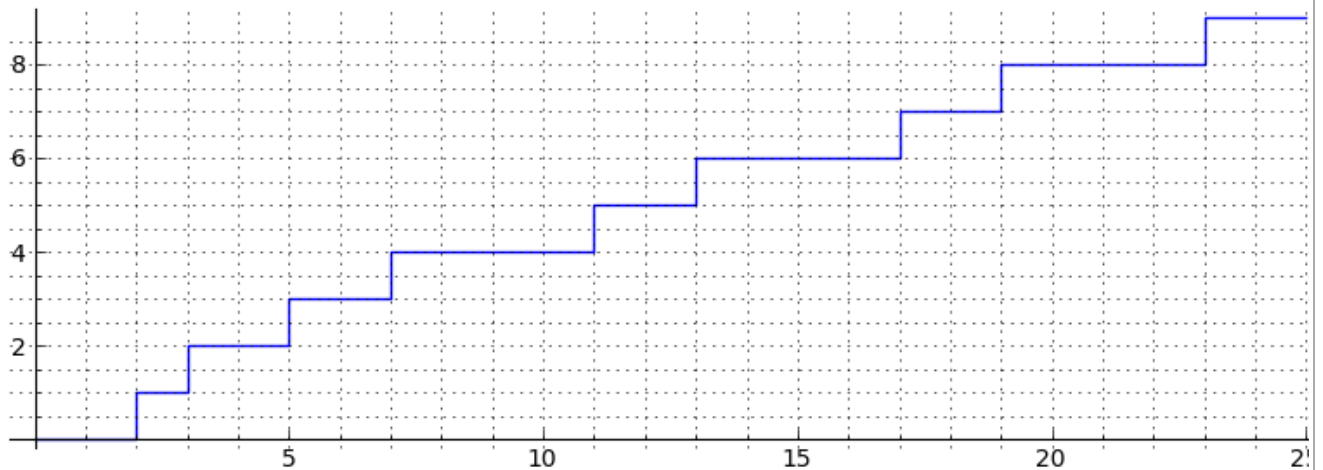
```
# Jon Bober - Rademacher's formula
time number_of_partitions(10^6)
```

```
147168498635822339863100476060989594348403048443914212533461274735166611\
741891861827633014887398359755584201537413060028809592938734712823227032\
784957800193278439607206422865904871302017097184076102567647986084690814\
282935670692978599129051989944549067221999782345287498297402228822985013\
676756629478188749468787900382469998819772920063206866873599666227381679\
826621348241720844663102742800191813219817718064651123454259502672842445\
259229678119344813999466473010574256435915479498918148528535137055139947\
671998169145902201559910195960141747407571543075002218489581520933901248\
173446944831932328015066538404299405417958775176129491624814247999880293\
650719525707448504757166277176390339144249511382329819526300833648982604\
583771220245530499638214460102853183200451904659196830278753741811848600\
061201685259354274198021504626724547323732184583342751252422746539913017\
407694128084740083154221799928607110833630331629828910244464969680539541\
679187548001085263677402202312846764691977502234856252074774184334365780\
153413070476197553037516970799928704028567784161934747236817177215404666\
4303121315630003467104673818
Time: CPU 0.03 s, Wall: 0.03 s
```

```
@interact
```

```
def _(n=(25..10000)):
    plot(prime_pi, 0, n, gridlines='minor').show(figsize=[8,3])
```

n 25



Graph Theory

"Sage's graph theory crushes anything I have met from the point of view of methods implemented. I would say: 'if you found a proprietary graph library and you are convinced that it is better for your needs than Sage, your license is on me.' But those licenses are really expensive :-D"

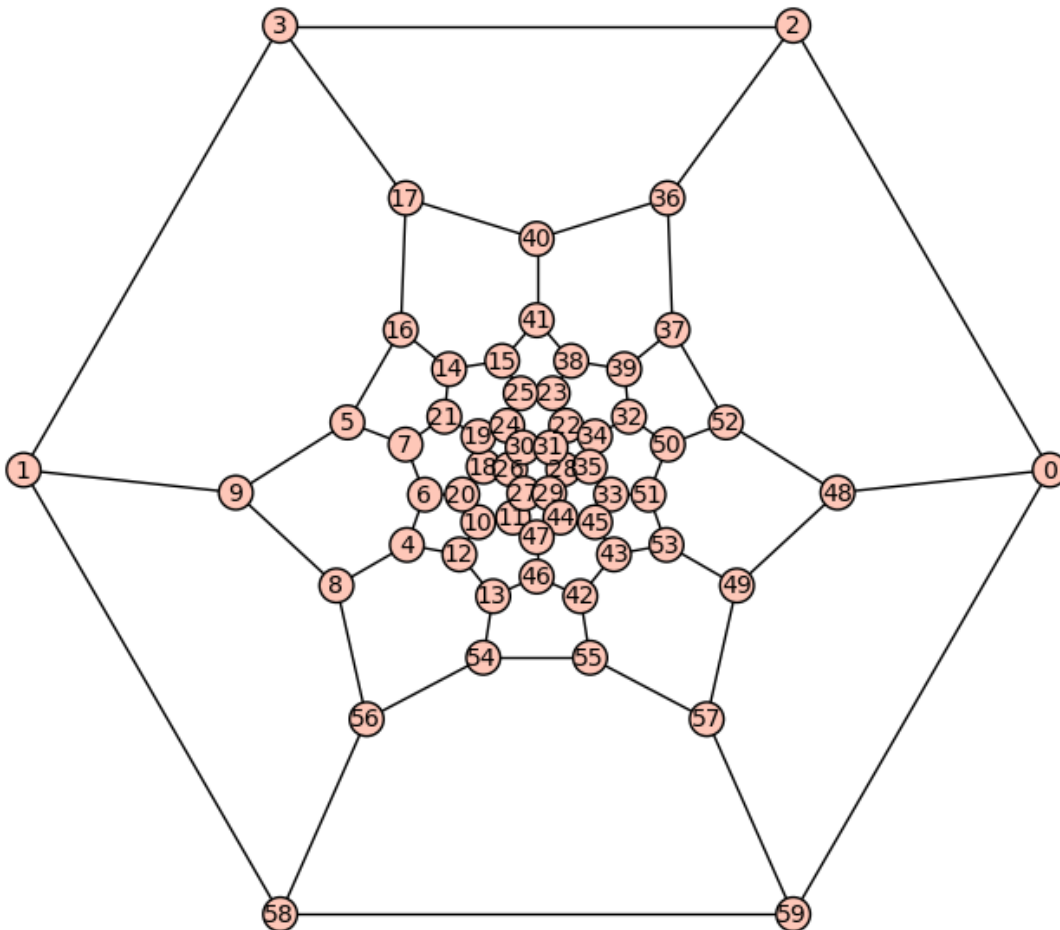
```
graphs.
```

```
Traceback (click to the left of this block for traceback)
```

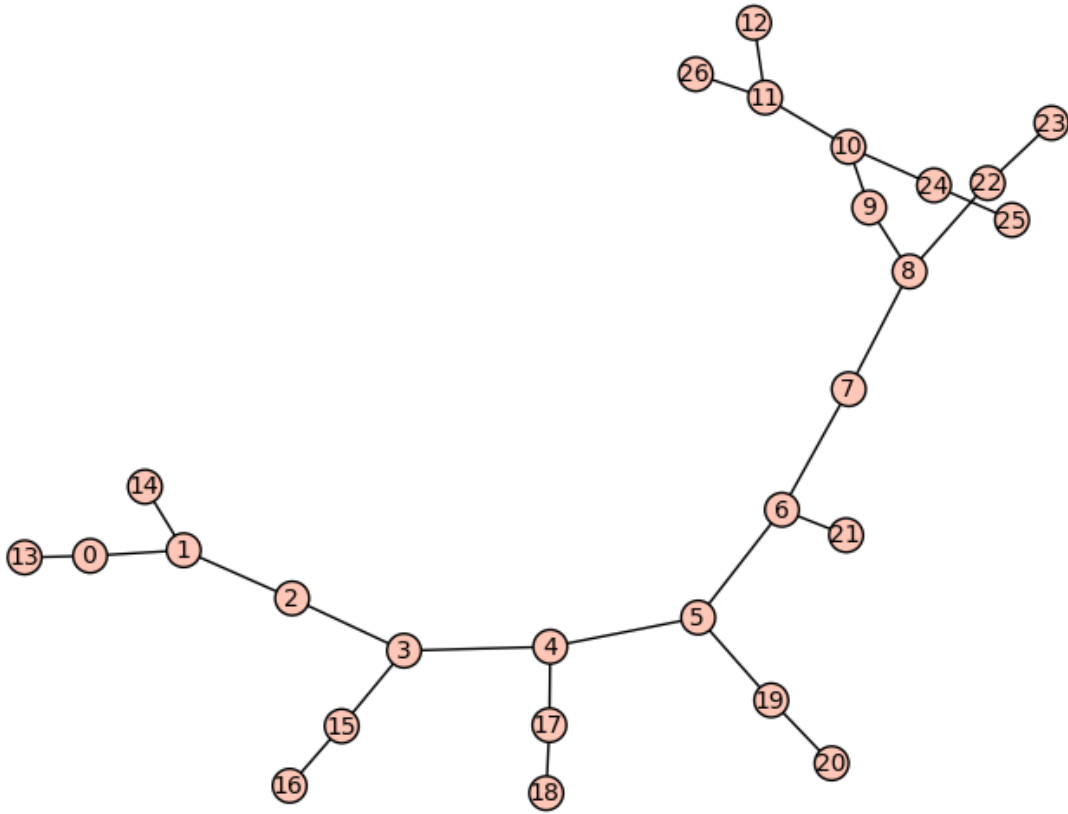
```
...
```

```
SyntaxError: invalid syntax
```

```
G = graphs.BuckyBall()
G.plot().show(figsize=8)
```



```
set_random_seed(1)
G = graphs.RandomLobster(8, .6, .3)
show(G, figsize=7)
```



```
G.automorphism_group()
```

```
Permutation Group with generators [(12,26)]
```

```
G.chromatic_number()
```

```
2
```

```
G.shortest_path(13,20)
```

```
[13, 0, 1, 2, 3, 4, 5, 19, 20]
```

Cython



- Smooth transition between Python and compiled C code.
- Make code that involves lots of manipulation of C-level data structures optimally fast
- Heavily used in scientific computing using Python.

```
def python_sum(n):  
    s = int(0)  
    for i in xrange(1, n+1):  
        s += i*i  
    return s
```

```
python_sum(3)
```

```
14
```

```
time python_sum(2*10^6)
```

```
2666668666667000000  
Time: CPU 0.18 s, Wall: 0.18 s
```

```
timeit('python_sum(2*10^6)')
```

```
5 loops, best of 3: 173 ms per loop
```

```
def python_sum2(n):  
    return sum(i*i for i in xrange(1,n+1))
```

```
time python_sum2(2*10^6)
```

```
2666668666667000000
Time: CPU 0.21 s, Wall: 0.21 s
```

```
%cython
def cython_sum(long n):
    cdef long long i, s = 0
    for i in range(1, n+1):
        s += i*i
    return s
```

[__mnt_sage...1_code_sage91_spyx.c](#) [__mnt_sage...ode_sage91_spyx.html](#)

```
cython_sum(3)
```

```
14L
```

```
time cython_sum(2*10^6)
```

```
2666668666667000000L
Time: CPU 0.00 s, Wall: 0.00 s
```

```
timeit('cython_sum(2*10^6)')
```

```
125 loops, best of 3: 2.79 ms per loop
```

```
165/.663
```

```
248.868778280543
```

```
%cython
def cython_sum2(long n):
    cdef long long i
    return sum(i*i for i in range(1,n+1))
```

[__mnt_sage...1_code_sage97_spyx.c](#) [__mnt_sage...ode_sage97_spyx.html](#)

```
time cython_sum2(2*10^6)
```

```
2666668666667000000
Time: CPU 0.22 s, Wall: 0.22 s
```

Of course, it is better to choose a different algorithm:

```
var('k, n')
factor(sum(k^2, k, 1, n))
```

```
1/6*(n + 1)*(2*n + 1)*n
```

```
def sum2(n):
    return n*(2*n+1)*(n+1)/6
```

```
sum2(2*10^6)
```

```
2666668666667000000
```


Even then, Cython provides a speedup:

```
%cython
def c_sum2(long long n):
    return n*(2*n+1)*(n+1)/6
```

[__mnt_sage..._code_sage104_spyx.c](#) [__mnt_sage...de_sage104_spyx.html](#)

```
c_sum2(3)
```

14L

```
c_sum2(2*10^6)
```

-407788678951258603L

```
n = 2*10^6
timeit('sum2(n)')
```

625 loops, best of 3: 2.29 μ s per loop

```
timeit('c_sum2(n)')
```

625 loops, best of 3: 145 ns per loop

```
2.01/.218
```

9.22018348623853

But at a cost!

```
c_sum2(2*10^6)          # WARNING: overflow -- it's just like C...
```

-407788678951258603L

```
n=2*10^6; n*(2*n+1)*(n+1) > 2^63
```

True

Solving Equations

Solve a *cubic equation*:

```
x = var('x'); show(solve(x^3 + x - 1==0, x)[0])
```

$$x = -\frac{1}{2} \left(i\sqrt{3} + 1 \right) \left(\frac{1}{18} \sqrt{3}\sqrt{31} + \frac{1}{2} \right)^{\frac{1}{3}} + \frac{-i\sqrt{3} + 1}{6 \left(\frac{1}{18} \sqrt{3}\sqrt{31} + \frac{1}{2} \right)^{\frac{1}{3}}}$$

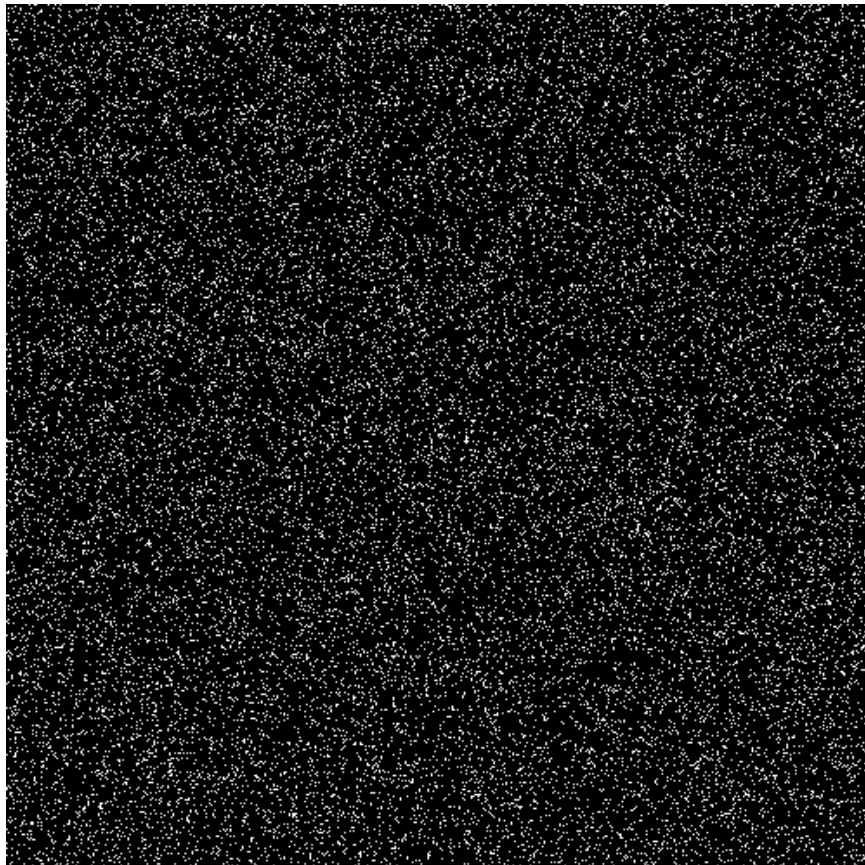
Solve a system of *two linear equations* with one unknown coefficient α :

```
var('alpha, y')
show(solve([3*x + 7*y == 2, alpha*x + 3*y == 8], x,y)[0])
```

$$\left[x = \frac{50}{7\alpha - 9}, y = \frac{2(\alpha - 12)}{7\alpha - 9} \right]$$

Solve a *system of 500 linear equations* exactly over the rational numbers:

```
n = 500
A = random_matrix(QQ, n, n, num_bound=10, den_bound=10)
v = random_matrix(QQ, n, 1, num_bound=10, den_bound=10)
A.visualize_structure()
```



```
# IML is used -- http://www.cs.uwaterloo.ca/~astorjoh/iml.html
time w = A \ v
```

```
Time: CPU 1.84 s, Wall: 1.85 s
```

```
print w.str()
```

```
WARNING: Output truncated!
full\_output.txt
```

```
[
-35170287606880290643755242068605585032607645790467696227510083012754014\
209552613191825054726554034468711333753512138553502268154641926254167854\
277993782242047975648765537789684779947163397757672378207320752013079903\
081977572402942342855631847634917334375567725200949287419814943328710246\
512541588734123916498194827140942961695474389949293216618783927383084043\
024220413380865093288810118537578983181053899461749302374084369069440792\
23742786833914611551409557282902026689939169134355578550571985033802542\
322986415496957988427958692956997351077992010024412064655355050589802049\
508358317945065384321675002925259301795806490140294417796752695156539524\
124805975616026523319266984818092269427921171858577830638088516230508666\
910804343311571401939976470565640553237263742498812830293003722157384120\
429185210964481771118171003582668463237232745438803646481729385618653719\
290569550431763124314799117129173800770979018877161538915171649899855408\
403216755234480553036017941777097575295161000394898303777475031991111715\
673283202172778539539244149073095868925297296767522098873014512743624494\
378981459885929562775838161397883770933575524969337769480812242031564479\
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089515861420506355106163849147350479391005608303463670362810424990990935\
874361744601474243543981369456229165217628375118975466429551361180619892\
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638947936431387935201327903186688186655093238897498819582538606101851386\
522984990931250079225108710736421552370716019268202256617920451380280400\
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919260114909352478204286878986163997358962243406000573598545315761469262\
512000898974810712781639389304729561277038985321773846773824153845455886\
996429984246385692956390637878107844843582575044052722770755619169229082\
572645352435097286438652915089536390796527241696946940261755243886211429\
377669055892193462392590930367528619341645259529861278091441114221626601\
873251672125619208172280955826296174872463796580072215353832080758361490\
836793768058045309403540953539624482725087280283849150746723268448986765\
756108006654291654308142745926766655027874073352324443620082948793916838\
38111946881427003740281908833698685908216732077/4067782875484212326663311\
617328670832844517668982653313360678139263119923699888566280990867348557\
356671155290956430174751875095360251105798206742735127104098165067814139\
492190954219009946130255863282988325207830875101159952490004624838536952\
540322189167758837780462767325132870033488208322866273391516599202161186\
024347828920828687409082348177640217448975984517612915621713638267535188\
696384621721767869207503970464394197565221463313299063259220623341268868\
453072277475331662210633800047415048666800872200330079604203595393425649\
170599073986150285231863177976366609996046241653617055408115480273443479\
448433183797541958284471397140590759568239535749614560724228213798534712\
659574179784679076480303433710416804886662134187828545003714490044646281\
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```

[full_output.txt](#)

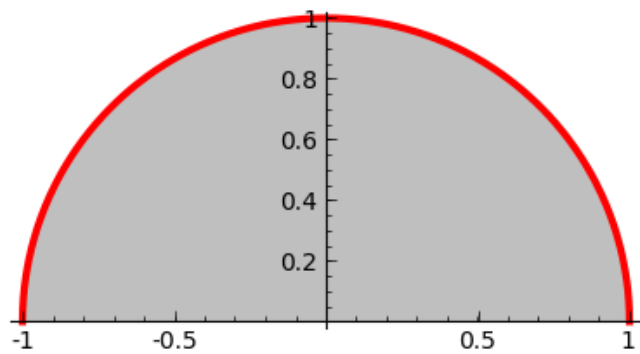
Symbolic Calculus

Symbolic Calculus makes use of **Maxima** and **Ginac** under the hood.

```

var('x')
f = sqrt(1 - x^2)
plot(f, thickness=3, color='red', aspect_ratio=1, fill=True, figsize=4)

```



```
var('t')
assume(t+1 > 0)
g = integrate(f, (x, -1, t)); show(g)
```

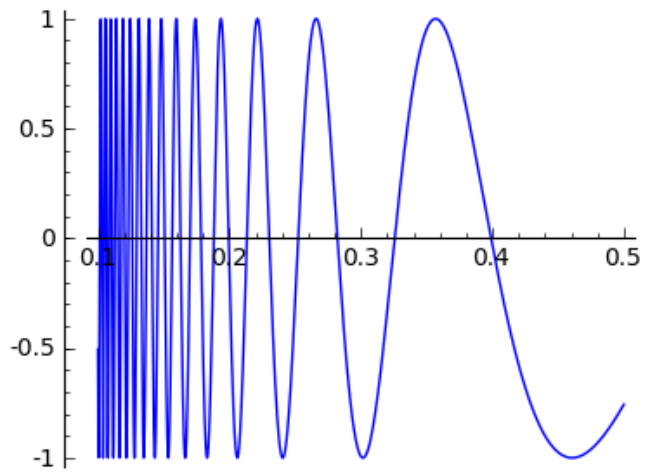
$$\frac{1}{4} \pi + \frac{1}{2} \sqrt{-t^2 + 1}t + \frac{1}{2} \arcsin(t)$$

```
show(g(t=1) - g(t=-1))
```

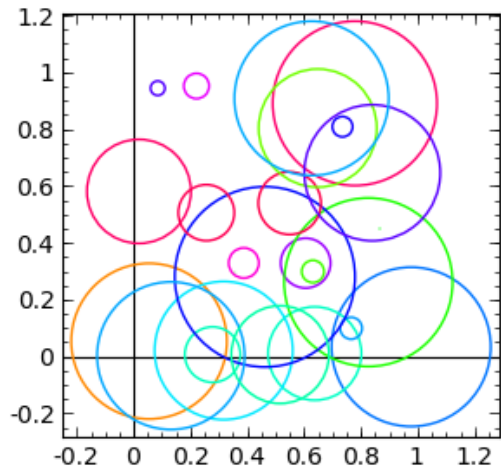
$$\frac{1}{2} \pi$$

Plotting in 2D

```
plot(sin(1/x^2), (x,.1,.5), figsize=4)
```



```
G = sum(circle((random(), random()), random()/3,
              color=hue(random())) for i in range(25))
G.show(aspect_ratio=1, frame=True, figsize=4)
```

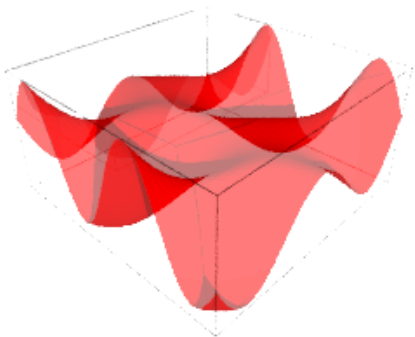


Plotting in 3D

```
f(x,y) = sin(x - y) * y * cos(x)
plot3d(f, (x,-3,3), (y,-3,3), opacity=.9, color='red', figsize=3)
```

[Get Image](#)

```
# user viewer='tachyon' on the ipad...  
f(x,y) = sin(x - y) * y * cos(x)  
plot3d(f, (x,-3,3), (y,-3,3), opacity=.9, color='red', figsize=3,  
viewer='tachyon')
```



```
G = sum(sphere((random(), random(), random()), random()/4,  
color=hue(random()), opacity=.6  
for i in range(20))  
G.show(aspect_ratio=1, frame=True, figsize=3)
```

[Get Image](#)



Plotting a 3D Model

See <http://www.davidson.edu/math/chartier/Starwars/>

```
# Yoda! (53,756 vertices)

from scipy import io
yoda = io.loadmat(DATA + 'yodapose.mat')

from sage.plot.plot3d.index_face_set import IndexFaceSet
V = yoda['V']; F3=yoda['F3']-1; F4=yoda['F4']-1
Y = IndexFaceSet(F3,V,color=Color('#444444')) +
IndexFaceSet(F4,V,color=Color('#007700'))
Y = Y.rotateX(-1)
Y.show(aspect_ratio=1, frame=False, zoom=1.2)
```

[Get Image](#)

Questions

????

If you made it this far, next try [the Sage Tutorial](#).

[evaluate](#)

