SAGE

Software for Algebra and Geometry Experimentation DEMO

Warm up

```
2 ^ 3
```
```
8
```
```
%python
2 ^ 3
```
```
1
```

SAGE has interfaces to most other computer algebra systems, commercial or free.

```
%maple
integrate(sin(x)*cos(x)+1/x,x)
```
```
1/2*sin(x)^2+ln(x)
```
```
magma('Factorization(-2006)')
```
```
[ <2, 1>, <17, 1>, <59, 1> ]
```

Matrices

```
m = matrix(5,5, range(25)); m
```
```
[ 0  1  2  3  4]
[ 5  6  7  8  9]
[10 11 12 13 14]
[15 16 17 18 19]
[20 21 22 23 24]
```
```
f = m.charpoly()
```
```
f
```
```
x^5 - 60*x^4 - 250*x^3
```
```
show(f)
```

$$x^5 - 60x^4 - 250x^3$$

```
show(factor(f))
```


$$(x^2 - 60x - 250) \cdot x^3$$

Some Graphics

Integer Factorization Tree

We illustrate integer factorization as a product of primes using the ``factor trees''...

```
F = factor_trees(factorial(10), rows=1, cols=1, font=14)
F.show(axes=False, figsize=[10,6])
```

A 3d Raytracer is Built into SAGE

The following is a picture of rational points on a rank 1 elliptic curve.

```
t = Tachyon(xres=700, yres=500, camera_center=(2,7,4), look_at=(2,0,0), raydepth=4)
t.light((10,3,2), 1, (1,1,1))
t.light((10,-3,2), 1, (1,1,1))
t.texture('black', color=(0,0,0))
t.texture('red', color=(1,0,0))
t.texture('grey', color=(.9,.9,.9))
t.plane((0,0,0),(0,0,1),'grey')
t.cylinder((0,0,0),(1,0,0),.01,'black')
t.cylinder((0,0,0),(0,1,0),.01,'black')
E = EllipticCurve('37a'); show(E)
P = E([0,0])
Q = P
n = 60
for i in range(n):
   Q = Q + P
   c = i/n + .1
   t.texture('r%s'%i,color=(float(i/n),0,0))
   t.sphere((Q[0], -Q[1], .01), .04, 'r%s'%i)
show(t)
```

$$y^2 + y = x^3 - x$$

Some Basic Calculus

This is work in progress with Bobby Moretti. SAGE includes Maxima, which is a

program that started at the AI lab at MIT in the 1960s!. It is capable of very sophisticated symbolic and numerical calculus. One can use it from SAGE via "maxima.[tab]", but we are writing new code so its functionality can be used naturally in SAGE without any knowledge of Maxima.

```
from sage.calculus.all import *

f = sin(x)*cos(x) + 1/x; show(f)
```

$$\sin(x) \cdot \cos(x) + \frac{1}{x}$$

```
g = f.integral(x); show(g)
```

$$\log(x) - \frac{\cos(x)^2}{2}$$

```
f = sin(x+y)*cos(x+z); show(f)
```

$$\sin(x+y) \cdot \cos(x+z)$$

```
h = f.trig_expand(); show(h)
```

$$\cos(x) \cdot \sin(y) + \sin(x) \cdot \cos(y) \cdot \cos(x) \cdot \cos(z) - \sin(x) \cdot \sin(z)$$

SAGE Worksheet: _scratch_ http://localhost:8002/_scratch___print__.html