Math 581g, Fall 2011, Homework 4

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Due: Friday, November 4, 2011

There are 5 problems, which all ask you to explicitly compute something about modular forms of **level 1**. You can use computer software (e.g., Sage) and there is no requirement that you understand why an answer you give is correct on this assignment ("the computer says so" is OK for full credit). Turn your solutions in Monday, November 4, 2011 in class (or via email). You may work with other people and can find the LaTeX of this file at `http://wstein.org/edu/2011/581g/hw/`. (I unfortunately will not have office hours 11:00am-3:15pm on Thursday, October 27, because I will be flying to Texas.)

1. What is the dimension of M_{40}? Of S_{40}? What is the dimension of S_{2012}?

2. (a) Write down an explicit basis for S_{40} where each q-expansion is given to precision $O(q^{10})$, i.e., give coefficient of q^9, then $+\cdots$.

   (b) Consider the q-expansion $f = \sum_{n=1}^{\infty} \mathrm{Tr}(T_n)q^n$, where T_n is the nth Hecke operator on S_{40}. Write f explicitly as a linear combination of your basis from the previous part of this problem.

3. What is the smallest integer B such that the Hecke operators $T_1, T_2, \ldots, T_B$ acting on S_{40} generate the Hecke algebra $\mathbf{T} = \mathbf{Z}[T_1, T_2, \ldots, T_B]$ as a **Z-module**?

4. Make a table that tabulates the characteristic polynomial of the Hecke opeator T_2 acting on the spaces $S_{12}, S_{14}, S_{16}, \ldots, S_{50}$.

5. Let $\tau(n)$ be the Ramanujan τ function, i.e., the coefficients of the modular form Δ.

   (a) Is $\tau(p) = 0$ for any $p < 10^6$?

   (b) What is the average of the numbers $\tau(p)$ over primes $p < 10^6$? [Hint: You may want to use the function `delta_qexp` to compute all $\tau(n)$ for $n < 10^6$ in a few seconds.]

   (c) What about the average of $\tau(p)/p^{11/2}$ over primes $p < 10^6$? (You may compute a floating point approximation.)