

Math 581b, Fall 2010, Homework 4

Due: Wednesday, October 27, 2010

Do the following problems, and turn them in by the beginning of class on Wednesday, October 27, 2010. There are 4 problems.

1. Suppose $K = \mathbf{Q}(\sqrt{a})$ is a quadratic number field with $a \in \mathbf{Z}$ squarefree. Let $D_K = \text{Disc}(K)$ and let \mathcal{O}_K be the ring of integers of K . Let c be any positive integer and let $\mathcal{O}_c = \mathbf{Z} + c\mathcal{O}_K$ be the subring of elements of the form $b + c\alpha$ for $b, c \in \mathbf{Z}$ and $\alpha \in \mathcal{O}_K$. Deduce a formula for $\text{Disc}(\mathcal{O}_c)$ in terms of D_K .
2. Compute (by any method at all) a basis for \mathcal{O}_K for $K = \mathbf{Q}(\sqrt{a})$ for $a = -7, -3, -1, 2, 3, 5$.
3. Let M be each of Sage, Magma, Pari, and Mathematica. Read/skim the chapter of the M reference manual about algebraic number fields of M and in each case, record some things that occur to you as you read. To make this easier, I've given links to all the manuals below:
 - (a) Sage: http://sagemath.org/doc/reference/number_fields.html
 - (b) Magma: <http://magma.maths.usyd.edu.au/magma/htmlhelp/text418.htm>
 - (c) Pari: Section 3.6 of <http://pari.math.u-bordeaux.fr/pub/pari/manuals/2.3.3/users.pdf>
 - (d) Mathematica: <http://reference.wolfram.com/mathematica/tutorial/AlgebraicNumberFields.html>

You can use Magma for free here: <http://magma.maths.usyd.edu.au/calc/>, Sage and Pari here: <http://uw.sagenb.org>, and Mathematica is I think available on zeno.

4. Consider the fields $\mathbf{Q}(\sqrt{p})$ for each prime number $p < 1000$. How many of these fields have class number 1? (You will need a computer to do this problem. Any software in problem 3 should be able to do this.)