Math 581b, Fall 2010, Homework 3

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Assigned: Wednesday, October 13, 2010

Do the following problems, and turn them in by the beginning of class on Wednesday, October 20, 2010. There are 5 problems.

1. Is the ideal $(2, \sqrt{-6})$ in the Dedekind domain $R = \mathbf{Z}[\sqrt{-6}]$ principal or not? (Prove your answer.) Recall that this came up during the lecture on Monday, Oct 11.
2. (a) What is the discriminant of $R = \mathbf{Z}[\sqrt{-6}]$?
   (b) What is the discriminant of the subring $S = \mathbf{Z}[5\sqrt{-6}]$ of R?
3. For which of the integers $n \in \{-1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ is there an elements $\alpha \in \mathbf{Z}[\sqrt{-6}]$ with norm n? (Recall that the norm of $a + b\sqrt{-6}$ is $(a + b\sqrt{-6})(a - b\sqrt{-6}) = a^2 + 6b^2$.)
4. Is there an element of $\mathbf{Q}(\sqrt{-6})$ of norm 3?
5. Let n be the number of ideals of $\mathbf{Z}[\sqrt{-6}]$ with norm ≤ 2010. Give an explicit upper bound on n, e.g., an integer M such that $n \leq M$. (It's fine if M is a wild overestimate of n, as long as you justify that your upper bound is valid.)